Design of Parallel and High Performance Computing
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Homework 7

Broadcast in the $\alpha\beta$ model

Task 1

The time taken to send a message of size s from one process to another is $T(s) = \alpha + s\beta$. In the $\alpha\beta$ model, if a process sends a message of size s at the time t it cannot send another message before $t + T(s)$. In the lecture we have seen the analysis of a broadcast over a binary and a binomial tree. However, we can also define a k-ary as well as a k-nomial tree broadcast.

In a k-ary tree broadcast every node forwards the received message to k children. A k-nomial tree is produced by forwarding the message to $k-1$ children every round, until all processes are reached.

- What is the runtime of a k-ary tree broadcast in the $\alpha\beta$ model if we assume small messages, i.e., $s = 1$?

Solution Each node has to send k messages, one for each of its k children. The last child will receive the message at: $k \cdot T(s)$. This is repeated for each level of the k-ary tree: $l \approx \lceil \log_k(P) \rceil$. In total, the broadcast time is:

$$T_{\text{bcast}}^{k\text{-ary}}(s) \approx k \cdot \lceil \log_k(P) \rceil \cdot T(s)$$

In a binomial tree (k=2), at every step each process that already received the data sends it to another process that did not receive it yet. In general, in a k-nomial tree, each process sends to $k-1$ other processes at each step.

- What is the runtime of a k-nomial tree broadcast in the $\alpha\beta$ model if we assume small messages, i.e., $s = 1$?

Solution At each stage, every node already having the data sends to other $k-1$ nodes not having the data. The number of nodes having received the data increases by a factor of k at each stage. Hence, we need $\lceil \log_k(P) \rceil$ stages to distribute the data to all the nodes.

$$T_{\text{bcast}}^{k\text{-nomial}}(s) = \lceil \log_k(P) \rceil \cdot (k-1) \cdot T(s)$$

Task 2

Assume P processors perform a broadcast operation by sending data in a ring (processor i receives from $(i-1+P) \mod P$ and sends to $(i+1) mod P$). Give a closed-form expression for the runtime of the broadcast in the $\alpha\beta$ model. Assume the data is of size m.

Solution The total number of communications that are needed to broadcast a message in a ring is $P-1$, where P is the number of involved processes.

$$T_{\text{bcast}}^{\text{ring}} = (P-1) \cdot T(m)$$